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ORIGINAL PAPER

A New Analytical Model to Evaluate Uncertainty of Wellbore Collapse Pressure Based on Advantageous Synergies of Different Strength Criteria

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Abstract
Considering the uncertainties of rock mechanical parameters, formation pressure and in situ stresses, the uncertainty of the wellbore collapse pressure should be evaluated. Before the uncertainty of evaluation, the collapse pressure model needs to be selected reasonably. In this paper, a new model was proposed to evaluate the collapse pressure, considering the advantageous synergies of different strength criteria. Especially, weight coefficients were introduced to represent the effect of different strength criteria on the collapse pressure, and were calculated by analytic hierarchy process. Then, an analytical method was proposed to address the uncertainty of the collapse pressure based on improved Rosenbluthe method, considering the new collapse pressure model. By means of the analytical method, the collapse pressure was obtained as the probability distribution under the condition that the uncertainties of input parameters were quantified based on well log data. More importantly, the analytical method was validated by Monte Carlo simulation. The results show that the probability distribution agrees very well between the analytical method and Monte Carlo simulation. Note that, the new collapse pressure model has the best matching for the probability distribution desired, which can be treated as the advantageous synergies of the new collapse pressure model.

Keywords Wellbore collapse pressure · Uncertainty · Probability distribution · Analytical method · Advantageous synergies

List of symbols

A	Coefficient related to the internal friction angle and the cohesion in Modified Lade criterion
A_1	Coefficient related to the internal friction angle and the cohesion in Mogi-Coulomb criterion
B	Coefficient related to the internal friction angle in Modified Lade criterion
B_1	Coefficient related to the internal friction angle in Mogi-Coulomb criterion
c	Cohesion
E_s	The static elastic module
g	The gravitational acceleration
I_1	The first stress invariant
J_2	The second deviatoric stress invariant
m	Coefficient related to the internal friction angle
k	Coefficient related to the internal friction angle and the cohesion
P_0	The drilling fluid pressure
P_p	The formation pressure
P_p^0	The hydrostatic pressure
x	The exponent constant
y^*	Coefficient used in improved Rosenbluthe method
y_i^+	Coefficient used in improved Rosenbluthe method
y_i^-	Coefficient used in improved Rosenbluthe method
z	Depth

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α	Biot's coefficient
$\Delta t_{c\_measured}$	The measured compressive sonic transit time by well logging
Δt_{c_normal}	The normal compressive sonic transit time in shale obtained from normal trend line
ε_x	The tectonic strain along the horizontal maximum stress direction
ε_y	The tectonic strains along the horizontal minimum stress direction
η_1	Weight coefficient, representing the effect of Mohr–Coulomb criterion on the collapse pressure
η_2	Weight coefficient, representing the effect of Drucker–Prager criterion on the collapse pressure
η_3	Weight coefficient, representing the effect of modified Lade criterion on the collapse pressure
η_4	Weight coefficient, representing the effect of Mogi–Coulomb criterion on the collapse pressure
v_s	The static Poisson's ratio
ρ	The bulk density
σ_H	The horizontal maximum in situ stress
σ_h	The horizontal minimum in situ stress
σ_v	The vertical minimum in situ stress
σ_r'	The radial effective stress
σ_θ'	The hoop effective stress
σ_z'	The vertical effective stress
φ	The internal friction angle

1 Introduction

During drilling, maintaining the wellbore stability is one of the most important tasks, mainly because the wellbore instability has a major effect on the drilling schedule and cost (Bradley 1979a, b; Aadnoy and Chenevert 1987). Currently, the wellbore stability is evaluated deterministically by the strength criterion, such as Mohr–Coulomb, Hoek–Brown, Lade, Drucker–Prager and Mogi–Coulomb criteria (Mohr 1900; Drucker and Prager 1952; Mogi 1967, 1971; Lade and Duncan 1975; Hoek and Franklin 1968, 1980, 1997; Al-Ajmi and Zimmerman 2005, 2006; Priest 2005; Zhang 2013; Zhang et al. 2015b). For this evaluation, however, two aspects of shortcomings can be found: (1) the collapse pressure has a larger difference when using different strength criteria, which means that the selection of the strength criterion is very important for the collapse-pressure prediction; (2) the collapse pressure is incorrect to be evaluated deterministically even if the strength criterion selected is reasonable, due to the uncertainties of rock mechanical parameters, together with formation pressure and in situ stresses. In view of these, a reasonable model of the collapse pressure was established first, and then the probabilistic analysis method was introduced to address the uncertainty of the collapse pressure.

Pioneering researchers have made important progresses to predict deterministically the wellbore stability using the strength criterion. Song and Haimson (1997) proposed a deterministic model to estimate wellbore breakout dimensions based the polyaxial strength criterion. Ewy (1999) proposed a deterministic model to evaluate the collapse pressure based on the modified Lade criterion. Colmenares and Zoback (2002) established a strength criterion of the intact rock using the test data from five different rocks, to predict the collapse pressure. Al-Ajmi and Zimmerman (2005, 2006) developed the Mogi–Coulomb criterion to predict the collapse pressure, where the maximum, minimum, and intermediate principal stresses were considered. Yi et al. (2006) used different models to calculate the reasonable drilling mud weight. Zhang and Zhu (2007) developed a 3D strength criterion to predict the collapse pressure. You (2009) established a true-triaxial model to model the wellbore stability. Zhang et al. (2010) proposed a model based on 3D Hoek–Brown criterion, to simulate the stability of vertical boreholes. Zhang et al. (2010) used five different rock strength criteria to analyze the minimum mud weights required for maintaining the wellbore stability, further recommending the 3D Hoek–Brown and Mogi–Coulomb criteria to predict the collapse pressure. Cai (2010) proposed a practical method to evaluate the tensile strength and Hoek–Brown strength parameters. Liu et al. (2012) established a deterministic model to predict the geomaterial failure based on the nonlinear Drucker–Prager and Matsuoka–Nakai criteria. Lee et al. (2012) developed 3D failure functions of Mohr–Coulomb and Hoek–Brown criteria, to evaluate the collapse pressure. Gholami et al. (2014) used the deterministic model to calculate the minimum drilling mud weight. Maleki et al. (2014) calculated the required safe drilling mud weight based on Mohr–Coulomb, Hoek–Brown and Mogi–Coulomb criteria. The results show that Mogi–Coulomb criterion is the most reasonable criterion, because of the consideration of the intermediate principal stress. Zhang et al. (2015a) developed an elastoplastic model to predict the collapse pressure for coal seam drilling based on Hoek–Brown criterion. Additionally, Wiebols and Cook (1968), Yudhbir et al. (1983), Pan and Hudson (1988), Bieniawski (1974), Desai and Salami (1987), McLean and Addis (1990) and Carter et al. (1991) also developed the corresponding models to evaluate the wellbore collapse. In these models above, different criteria have been proposed to predict the collapse pressure, respectively, having different advantages. However, the collapse pressure does not show the consistency for different strength criteria, which means that the collapse pressure obtained by the existing strength criteria is not the true collapse pressure. In other words, the true collapse pressure is very difficult to be found by single strength criteria. In view of this, a new model of the wellbore collapse pressure needs to be established to find the true collapse pressure possibly, depending on the advantageous synergies of the different strength criteria.

Considering that deterministic models can not deal with the uncertainty of the input parameter, the probabilistic analysis approach was introduced by some researchers, to account for the uncertainties of rock mechanical parameters, formation pressure and in situ stresses. Morita (1995) proposed a probabilistic model of borehole stability, considering the uncertainties of rock strength, shale swelling, in situ stresses, and pore pressure. Dumans (1995) evaluated the uncertainty of the wellbore collapse and the tensile failure using Monte Carlo simulation and fuzzy sets method. Ottesen et al. (1999) evaluated the uncertainties of the collapse pressure based on quantitative risk assessment, where limit state functions were defined as functions of wellbore trajectory and geometry. The result showed that the probability of success was closely related to the drilling fluid density. Liang (2002) proposed a prediction for the formation pressure and the fracture pressure using quantitative risk assessment, to obtain the corresponding distributions. De Fontoura et al. (2002) proposed three analytical models to estimate the uncertainty of the collapse pressure based on reliability indexes. Three analytical methods showed a good agreement to Monte Carlo simulation. Moos et al. (2003) used quantitative risk assessment to calculate the probability of avoiding the wellbore collapse for a given mud weight, considering the uncertainty of the input parameter. Meanwhile, a sensitivity analysis was performed to determine key input parameters. Sheng et al. (2006) developed a geostatistical approach to estimate the uncertainty of the wellbore stability, where Monte Carlo analysis technique was specially incorporated into a numerical model. Using the geostatistical approach, the safe mud weight was obtained with a distribution range. Luis et al. (2009) used Monte Carlo simulation to analyze the lower and upper limits of the collapse pressure, considering the uncertainties of the mechanical properties, the initial formation pressure and the in situ stresses. Al-Ajmi and Al-Harthy (2010) used Monte Carlo simulation to capture the uncertainty of input variables, further calculating the mud weight as a probability distribution. The authors considered that the proposed probabilistic model could quantify the effect of the input uncertainty on the output uncertainty. Aadnøy (2011) provided a new method to investigate the collapse pressure, taking the uncertainty of the input data into account. The paper thought that minimizing the range of the input data was the most important way to reduce the uncertainty of the model output. Mostafavi et al. (2011) proposed a model to evaluate the uncertainty of the collapse pressure, where quantitative risk assessment was used to calculate the probability of avoiding the wellbore instability. Udegbunam et al. (2014) used Monte Carlo simulation to estimate the uncertainty of the collapse pressure, taking the uncertainties of in situ stresses, rock strength, and formation pressure as the input parameters into account. Gholami et al. (2015) analyzed the effect of uncertainty of the input parameter on the safe mud weight window (MWW) using quantitative risk assessment, where different failure criteria were considered as limit state equations. The results showed that the maximum horizontal stress was the most sensitive parameter for establishing MWW. Niño (2016) evaluated the uncertainty of the collapse pressure by Monte Carlo simulation, considering the uncertainty of the input parameter.

Through the investigation above, it can be found that almost all uncertainties analyses of the collapse pressure are based on quantitative risk assessment and Monte Carlo simulation, except for the analytical methods of De Fontoura et al. (2002) and Aadnøy (2011). In fact, the key technique of quantitative risk assessment is also Monte Carlo simulation, which can be found in the work of Moos et al. (2003). In other words, Monte Carlo simulation is the most commonly used method for evaluating the uncertainty of the collapse pressure. However, Monte Carlo simulation is rather demanding and time-consuming due to plenty of simulation times (commonly exceeding 10,000 times), which greatly limits the field engineering application of this method (De Fontoura et al. 2002). In such case, the analytical method is still playing a necessary role to evaluate the uncertainty of the collapse pressure, as an important supplement to Monte Carlo simulation. Currently, two analytical models were developed by De Fontoura et al. (2002) and Aadnøy (2011). However, these two analytical models have a lower accuracy and a wider distribution range for the collapse-pressure prediction, because the calculation method is too simple.

Comprehensively analyzing the previous content, two problems need to be solved: (1) a new model of the collapse pressure needs to be established to play the advantageous synergies of different strength criteria; (2) an analytical method needs to be proposed to reduce the calculation time and enhance the calculation efficiency for the uncertainty analysis of the collapse pressure, compared to Monte Carlo simulation. In this paper, an analytical method was proposed to evaluate the uncertainty of the collapse pressure based on a new model of the collapse pressure. Using the proposed analytical method, the collapse pressure was obtained as the probability distribution function. Compared to the analytical models from De Fontoura et al. (2002) and Aadnøy (2011), the proposed model can provide a higher accuracy for the collapse pressure.

2 Uncertainty Analysis for Wellbore Collapse Pressure

2.1 New Collapse Pressure Model

According to elastic mechanics, the principle stresses on the wellbore surface can be expressed as Eq. (1):

$$\begin{cases}\sigma_r' = P_0 - \alpha P_p \\ \sigma_\theta' = 3\sigma_H - \sigma_h - P_0 - \alpha P_p \\ \sigma_z' = \sigma_v + 2\nu(\sigma_H - \sigma_h) - \alpha P_p\end{cases} \tag{1}$$

where σ_r', σ_θ', and σ_z' are ,respectively, the radial, hoop and vertical effective stresses, σ_H and σ_h represent the horizontal maximum and minimum in situ stresses, P_0 is the drilling fluid pressure, P_p is the formation pressure, α is Biot's coefficient and is assumed to be 1 in this analysis.

Substituting Eq. (1) into different strength criteria, e.g., Mohr–Coulomb criterion (Mohr 1900), Drucker–Prager criterion (Drucker and Prager 1952), modified Lade criterion (Lade and Duncan 1975) and Mogi–Coulomb criterion (Mogi 1967), the wellbore collapse pressure can be predicted.

2.1.1 Collapse Pressure Model Based on Mohr–Coulomb Criterion

Mohr–Coulomb criterion is the simplest one for evaluating the rock failure, and is commonly expressed as Eq. (2):

$$\sigma_1' = \sigma_3' \frac{1+\sin\varphi}{1-\sin\varphi} + \frac{2c\cos\varphi}{1-\sin\varphi} \tag{2}$$

where φ is the internal friction angle, c is the cohesion.

Substituting Eq. (1) into Eq. (2), the wellbore collapse pressure P_{wc}^{I} can be written as Eq. (3):

$$P_{wc}^{I} = \frac{(3\sigma_H - \sigma_h)(1-\sin\varphi) + 2\alpha P_p \sin\varphi - 2c\cos\varphi}{2} \tag{3}$$

2.1.2 Collapse Pressure Model Based on Drucker–Prager Criterion

Drucker–Prager criterion considers the effect of the intermediate principal stress on the collapse pressure. The empirical equation can be given in terms of the effective principle stresses, as shown in Eq. (4):

$$\sqrt{J_2} = mI_1 + k \tag{4}$$

where I_1 and J_2 are expressed as Eqs. (5) and (6), respectively, representing the first stress invariant and the second deviatoric stress invariant.

$$I_1 = \sigma_1' + \sigma_2' + \sigma_3' \tag{5}$$

$$J_2 = \frac{(\sigma_1' - \sigma_2')^2 + (\sigma_2' - \sigma_3')^2 + (\sigma_3' - \sigma_1')^2}{6} \tag{6}$$

and m, k are two coefficients and are related to the internal friction angle and the cohesion, seeing Eq. (7):

$$m = \frac{2\sin\varphi}{\sqrt{3}(3-\sin\varphi)}, \quad k = \frac{6c\cos\varphi}{\sqrt{3}(3-\sin\varphi)}, \tag{7}$$

Substituting Eq. (1) into Eq. (4), the collapse pressure P_{wc}^{II} can be expressed as Eq. (8):

$$P_{wc}^{II} = \frac{-b_1 - \sqrt{b_1^2 - 4a_1c_1}}{2a_1} \tag{8}$$

where $a_1 = 6$, $b_1 = 6\sigma_h - 18\sigma_H$,

$$\begin{aligned} c_1 = {} & \left[2(9 - 6\nu + 4\nu^2)\sigma_H + 4(4\nu - 4\nu^2 - 3)\sigma_h + (8\nu - 6)\sigma_v\right]\sigma_H \\ & + \left[(8\nu^2 - 4\nu + 2)\sigma_h + (2 - 8\nu)\sigma_v\right]\sigma_h \\ & + 2\sigma_v^2 - 6\left\{m\left[3\sigma_H - \sigma_h + \sigma_v + 2\nu(\sigma_H - \sigma_h) - 3\alpha P_p\right] + k\right\}^2 \end{aligned}$$

2.1.3 Collapse Pressure Model Based on Modified Lade Criterion

Modified Lade criterion is an extension of Lade and Duncan criterion, and can be expressed as Eq. (9):

$$\begin{aligned} & \left[(\sigma_1' + A) + (\sigma_2' + A) + (\sigma_3' + A)\right]^3 \\ & = (27 + B)(\sigma_1' + A)(\sigma_2' + A)(\sigma_3' + A) \end{aligned} \tag{9}$$

where A and B are expressed as Eq. (10):

$$A = \frac{c}{\tan\varphi}, \quad B = \frac{4\tan^2\varphi(9 - 7\sin\varphi)}{1 - \sin\varphi}, \tag{10}$$

Substituting Eq. (1) into Eq. (9), the collapse pressure P_{wc}^{III} can be expressed as Eq. (11):

$$P_{wc}^{III} = \frac{-b_2 - \sqrt{b_2^2 - 4a_2c_2}}{2a_2} \tag{11}$$

where $a_2 = (27 + B)\left[\sigma_v + 2\nu(\sigma_H - \sigma_h) - \alpha P_p + A\right]$,

$b_2 = -(27 + B)(3\sigma_H - \sigma_h)\left[\sigma_v + 2\nu(\sigma_H - \sigma_h) - \alpha P_p + A\right]$,

$$\begin{aligned} c_2 = {} & \alpha P_p(27 + B)(3\sigma_H - \sigma_h - \alpha P_p)\left[\sigma_v + 2\nu(\sigma_H - \sigma_h) - \alpha P_p\right] \\ & - A(27 + B)\left\{\left[\sigma_v + 2\nu(\sigma_H - \sigma_h) - 2\alpha P_p\right](3\sigma_H - \sigma_h - \alpha P_p) \right. \\ & \left. - \left[\sigma_v + 2\nu(\sigma_H - \sigma_h) - \alpha P_p\right]\alpha P_p\right\} \\ & - A^2(27 + B)\left[3\sigma_H - \sigma_h + \sigma_v + 2\nu(\sigma_H - \sigma_h) - 3\alpha P_p + A\right] \\ & + \left[3\sigma_H - \sigma_h + \sigma_v + 2\nu(\sigma_H - \sigma_h) - 3\alpha P_p + 3A\right]^3 \end{aligned}$$

2.1.4 Collapse Pressure Model Based on Mogi–Coulomb Criterion

Mogi–Coulomb criterion is a modification of Mohr–Coulomb criterion, and is expressed as follow:

$$\tau_{oct} = A_1 + B_1\sigma_{m,2}' \tag{12}$$

where

$$\tau_{\text{oct}} = \frac{1}{3}\sqrt{\left(\sigma_1' - \sigma_2'\right)^2 + \left(\sigma_2' - \sigma_3'\right)^2 + \left(\sigma_3' - \sigma_1'\right)^2}, \quad \sigma_{m,2}' = \frac{\sigma_1' + \sigma_3'}{2} \tag{13}$$

$$A_1 = \frac{2\sqrt{2}}{3} c \cos\varphi, \quad B_1 = \frac{2\sqrt{2}}{3} \sin\varphi \tag{14}$$

Substituting Eq. (1) into Eq. (12), the collapse pressure $P_{\text{wc}}^{\text{IV}}$ can be expressed as Eq. (15):

$$P_{\text{wc}}^{\text{IV}} = \frac{-b_3 - \sqrt{b_3^2 - 4a_3c_3}}{2a_3} \tag{15}$$

where $a_3 = 3$, $b_3 = 3\sigma_{\text{h}} - 9\sigma_{\text{H}}$,

$$\begin{aligned} c_3 = {} & \left(3\sigma_{\text{H}} - \sigma_{\text{h}} - \alpha P_{\text{p}}\right)^2 + \left[\sigma_{\text{v}} + 2v\left(\sigma_{\text{H}} - \sigma_{\text{h}}\right) - \alpha P_{\text{p}}\right]^2 \\ & - 2v\left(\sigma_{\text{H}} - \sigma_{\text{h}}\right)\left(3\sigma_{\text{H}} - \sigma_{\text{h}} - 2\alpha P_{\text{p}}\right) \\ & + 2\alpha P_{\text{p}}\left(3\sigma_{\text{H}} - \sigma_{\text{h}} + \sigma_{\text{v}}\right) - \frac{9}{2}\left[A_1 + \frac{B_1}{2}\left(3\sigma_{\text{H}} - \sigma_{\text{h}} - 2\alpha P_{\text{p}}\right)\right]^2 \\ & - 3\sigma_{\text{H}}\sigma_{\text{v}} + \sigma_{\text{h}}\sigma_{\text{v}} - 3\alpha^2 P_{\text{p}}^2 \end{aligned}$$

2.1.5 New Model of Collapse Pressure

For the current collapse pressure model, it is generally accepted that Mohr–Coulomb criterion over-predicts the collapse pressure while Drucker–Prager criterion under-predicts the collapse pressure. That is to say, the true collapse pressure should be less than the one of Mohr–Coulomb criterion, while more than the one of Drucker–Prager criterion. In such a case, different strength criteria should play the advantageous synergies to find the true collapse pressure, i.e., the true collapse pressure should be analyzed by combining the existing different strength criteria and the corresponding weight coefficient. In view of this, a new model was established to evaluate the wellbore collapse pressure, as shown in Eq. (16). From the equation form point of view, the new model has a more comprehensive coverage for the collapse pressure.

$$P_{\text{wc}} = \eta_1 P_{\text{wc}}^{\text{I}} + \eta_2 P_{\text{wc}}^{\text{II}} + \eta_3 P_{\text{wc}}^{\text{III}} + \eta_4 P_{\text{wc}}^{\text{IV}} \tag{16}$$

where η_1, η_2, η_3, and η_4 are the weight coefficients, representing the effects of Mohr–Coulomb, Drucker–Prager, modified Lade, and Mogi–Coulomb criteria on the collapse pressure, respectively.

The new model can transform into the existing strength criteria by changing the weight coefficient. For example, the new collapse pressure model is identical to the collapse pressure model developed by Mohr–Coulomb criterion, considering $\eta_1 = 1$, $\eta_2 = 0$, $\eta_3 = 0$, $\eta_4 = 0$. Similarly, $\eta_1 = 0$, $\eta_2 = 0$, $\eta_3 = 0$, $\eta_4 = 1$ means that the new collapse pressure model is similar to the collapse pressure model developed by Mogi–Coulomb criterion. Overall, the new model of the collapse pressure can involve comprehensively Mohr–Coulomb, Drucker–Prager, modified Lade and Mogi–Coulomb criteria, because of introduction of weight coefficients.

Note that, the weight coefficients can be determined based on analytic hierarchy process (AHP), by comparing the relative importance of different strength criterion. In the AHP, the values of the pairwise comparisons were used to describe the relative important degree of different strength criterion. The available values for the pairwise comparisons are members of the set: {9, 8, 7, 6, 5, 4, 3, 2, 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9}. Especially, the values of the pairwise comparisons are approximately objective values, but instead of arbitrary subjective values given by experts. In such case, the collapse pressure would be approximately constant even if two experts try to determine the collapse pressure. According to the above analysis, it can be seen that determining the values of the pairwise comparisons is the most important step for weight coefficients. In view of this, the following operations were performed to determine the reliable values for the pairwise comparisons:

1. n sets of rock samples from the drilling site were tested by the indoor experiments to obtain the failure stresses. The test data of the failure stresses were fitted by different strength criteria, including Mohr–Coulomb criterion, Drucker–Prager criterion, modified Lade criterion, and Mogi–Coulomb criterion.
2. Counting the number of rock samples that the test data coincide well with Mohr–Coulomb criterion and marking it as n_1. In the same way, counting the numbers of rock samples that satisfy Drucker–Prager criterion, modified Lade criterion, Mogi–Coulomb criterion and marking them as n_2, n_3, n_4, respectively. Especially, the standard that the test data coincide well with the strength criteria is that the relative error between the test data and the fitted data is within 3%.
3. The reliable values for the pairwise comparisons can be obtained by calculating the ratios between n_1, n_2, n_3, and n_4. The reliable values for the pairwise comparisons would play a very important guideline for different experts.

In addition to the above analysis, the weight coefficients should also satisfy the basic engineering experience (Zhang et al. 2010), i.e., Drucker–Prager criterion has the smallest weight coefficient, followed by modified Lade criterion and Mohr–Coulomb criterion, and Mogi–Coulomb criterion has the largest weight coefficient.

2.2 Uncertainty Analysis Based on Improved Rosenbluthe Method

In this uncertainty analysis, the in situ stresses σ_{H}, σ_{h}, and σ_{v}, the formation pressure P_{p} and the rock mechanical parameters φ and c were treated as input variables. Improved Rosenbluthe method was introduced to calculate mean and standard deviation of the collapse pressure, further determining the probability distribution of the collapse pressure.

Using improved Rosenbluthe method (Rosenblueth 1985; Cui et al. 1998), three function values were first calculated, namely, y^*, y_i^+, and y_i^-, seeing Eqs. (17)–(19).

$$y^* = G(\mu_{X_1}, \dots, \mu_{X_i}, \dots, \mu_{X_n}) \tag{17}$$

$$y_i^+ = G(\mu_{X_1}, \dots, \mu_{X_i} + \sigma_{X_i}, \dots, \mu_{X_n}) \tag{18}$$

$$y_i^- = G(\mu_{X_1}, \dots, \mu_{X_i} - \sigma_{X_i}, \dots, \mu_{X_n}) \tag{19}$$

According to Eqs. (18) and (19), two coefficients, μ_i, σ_i, were defined as follow:

$$\mu_i = \frac{y_i^+ + y_i^-}{2} \tag{20}$$

$$\sigma_i = \frac{\left|y_i^+ - y_i^-\right|}{2} \tag{21}$$

Additionally, the function values of y_{ij}^+ and y_{ij}^- were determined based on Eqs. (22) and (23):

$$y_{ij}^+ = G\left(\mu_{X_1}, \dots, \mu_{X_i} + \sigma_{X_i}, \dots \mu_{X_j} + \sigma_{X_j}, \dots, \mu_{X_n}\right) \tag{22}$$

$$y_{ij}^- = G\left(\mu_{X_1}, \dots, \mu_{X_i} - \sigma_{X_i}, \dots \mu_{X_j} - \sigma_{X_j}, \dots, \mu_{X_n}\right) \tag{23}$$

Then, the coefficients, μ_{ij}, $\Delta\mu_i$, and $\Delta\mu_{ij}$, were calculated using Eqs. (24)–(26):

$$\mu_{ij} = \frac{y_{ij}^+ + y_{ij}^-}{2} \ (i < j) \tag{24}$$

$$\Delta\mu_i = \mu_i - y^* \tag{25}$$

$$\Delta\mu_{ij} = \mu_{ij} - y^* - \Delta\mu_i - \Delta\mu_j \tag{26}$$

Finally, mean and standard deviation of the function composed by random variables can be obtained as follow:

$$\begin{cases} \mu_Y = y^* + \sum_{i=1}^{n} \Delta\mu_i \\ \sigma_Y^2 = \sum_{i=1}^{n} \sigma_i^2 + 2\sum_{i=1}^{n} \left(\Delta\mu_i\right)^2 + \sum_{i<j} \left(\Delta\mu_{ij}\right)^2 \end{cases} \tag{27}$$

According to the conclusion from Cui et al. (1998), improved Rosenbluthe method shows a good accuracy with second-order Taylor expansion for predicting mean and standard deviation of the function composed by random variables.

3 Uncertainty Quantification for Input Parameter

Plenty of samples need to be generated to quantify comprehensively the uncertainty of input parameters. For the sample generation, three kinds of methods can be usually considered. One is the measured data as the samples, including the indoor experiment data and the field data. However, these measured data are often sparse and rare, and are difficult to exhibit the probability distribution of the input parameter. The second method is to use the sampling technique to generate the samples, by specifying the mean, the maximum value and the minimum value of the input parameter. By means of this method, the sample space generated is large enough. However, the sample data may be not used in practice engineering, once the input parameter (e.g., the mean, the maximum value and the minimum value) can not accurately describe the actual geological condition. The third method is based on well log data. Log data can provide the sufficient samples, considering that log data can be transformed into the input parameter, such as the in situ stresses, the formation pressure and the rock mechanical property. From this point of view, well log data are the most suitable source of information to quantify the uncertainty of the input parameter. In the following analysis, well log data were used to generate the samples of random variables, quantifying the uncertainty of each input parameter.

3.1 Uncertainty Quantification for Formation Pressure

The formation pressure can be evaluated based on well log data, by introducing Eaton's method (Eaton 1975). Eaton gave the empirical equation of the formation pressure based on the compressive sonic transit time, seeing Eq. (28):

$$P_{\mathrm{p}} = \sigma_{\mathrm{v}} - \left(\sigma_{\mathrm{v}} - P_{\mathrm{p}}^0\right)\left(\frac{\Delta t_{\mathrm{c\_normal}}}{\Delta t_{\mathrm{c\_measured}}}\right)^x \tag{28}$$

where P_{p} and P_{p}^0 are the pore pressure and the hydrostatic pressure, respectively, $\Delta t_{\mathrm{c\_measured}}$ stands for the measured compressive sonic transit time by well logging, $\Delta t_{\mathrm{c\_normal}}$ is the normal compressive sonic transit time in shale obtained from normal trend line, x represents the exponent constant and is assumed to be originally 3, σ_{v} is the overburden pressure, and can be expressed as Eq. (29):

$$\sigma_{\mathrm{v}} = \int_0^z \rho g \mathrm{d}z \tag{29}$$

where ρ is the bulk density and can be obtained by sonic logs, g is the gravitational acceleration, and z is the depth.

In this analysis, well log data were from shale layer of LZT_02 well in XinJiang oil field, China. Using these log data, a total of 1400 samples were generated for the formation pressure. Subsequently, the samples obtained were quantified, and uncertainty was identified as a probability distribution function, as shown in Fig. 1.

As seen from Fig. 1a, the formation pressure has a variation of [23.2 MPa, 24.8 MPa] for shale layer of LZT_02 well, and 1386 samples were within this interval. By collecting the data number of each 0.1 MPa, the probability distribution was fitted and plotted in Fig. 1b. Obviously, the formation pressure shows a normal distribution.

3.2 Uncertainty Quantification for In Situ Stresses

To quantify the uncertainty of in situ stresses, the relation was first given between well log data and the in situ stresses (Blanton and Olson 1997), seeing Eq. (30).

$$\begin{cases} \sigma_{\mathrm{H}} = \dfrac{v_{\mathrm{s}}}{1-v_{\mathrm{s}}}\sigma_{\mathrm{v}} + \dfrac{1-2v_{\mathrm{s}}}{1-v_{\mathrm{s}}}\alpha P_{\mathrm{P}} + \dfrac{E_{\mathrm{s}}}{1-v_{\mathrm{s}}^2}\varepsilon_{\mathrm{x}} + \dfrac{v_{\mathrm{s}}E_{\mathrm{s}}}{1-v_{\mathrm{s}}^2}\varepsilon_{\mathrm{y}} \\ \sigma_{\mathrm{h}} = \dfrac{v_{\mathrm{s}}}{1-v_{\mathrm{s}}}\sigma_{\mathrm{v}} + \dfrac{1-2v_{\mathrm{s}}}{1-v_{\mathrm{s}}}\alpha P_{\mathrm{P}} + \dfrac{E_{\mathrm{s}}}{1-v_{\mathrm{s}}^2}\varepsilon_{\mathrm{y}} + \dfrac{v_{\mathrm{s}}E_{\mathrm{s}}}{1-v_{\mathrm{s}}^2}\varepsilon_{\mathrm{x}} \end{cases} \tag{30}$$

where E_s is the static elastic module, v_s is the static Poisson's ratio, ε_x and ε_y are the tectonic strains along the horizontal maximum and minimum stress directions, respectively, and can be solved deterministically by the hydraulic fracturing data.

Based on well log data, the total of 1400 samples were generated. Then, uncertainties were quantified for the maximum, minimum, and intermediate principal stresses, see Figs. 2, 3 and 4.

Through uncertainty quantification, in situ stresses show a clear normal distribution, regardless of the maximum, minimum or intermediate principal stress, which can be seen in Figs. 2, 3 and 4. Note that, there are 1378, 1391, and 1388 samples within the intervals of the maximum principal stress, the intermediate principal stress, the minimum principal stress, respectively.

3.3 Uncertainty Quantification for Mechanical Parameters

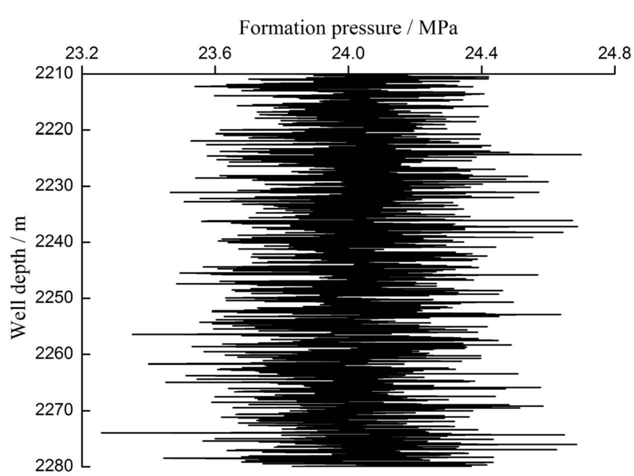
Manohar (1999) established Eq. (31) to obtain the internal friction angle φ (°) and the cohesion c (MPa), using the compressive sonic velocity V_p (km/s).

$$\sin\varphi = \frac{V_{\mathrm{p}}-1}{V_{\mathrm{p}}+1}, \quad c = \frac{5\left(V_{\mathrm{p}}-1\right)}{\sqrt{V_{\mathrm{p}}}} \tag{31}$$

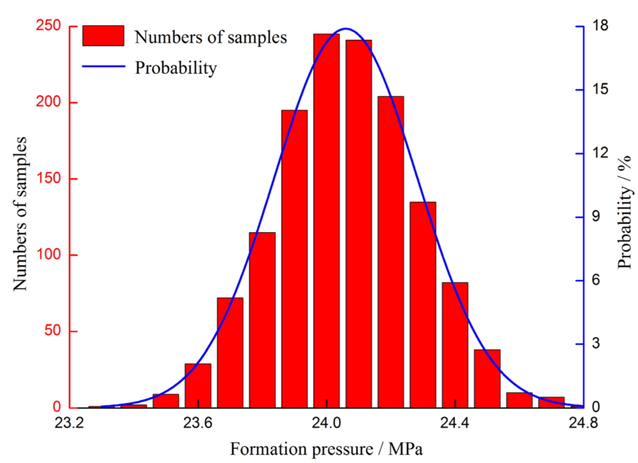
Using well log data mentioned previously, the total of 1400 samples were generated. Then, uncertainties were evaluated for the cohesion and the internal friction angle, being shown in Figs. 5 and 6. Note that, there are 1380 and 1383 samples within the intervals of the cohesion and the internal friction angle, respectively.

More importantly, the statistical properties of all the input parameters were summarized in Table 1.

Using the results listed in Table 1, the uncertainty of the collapse pressure can be evaluated. The detailed procedure of calculations is shown in Fig. 7.

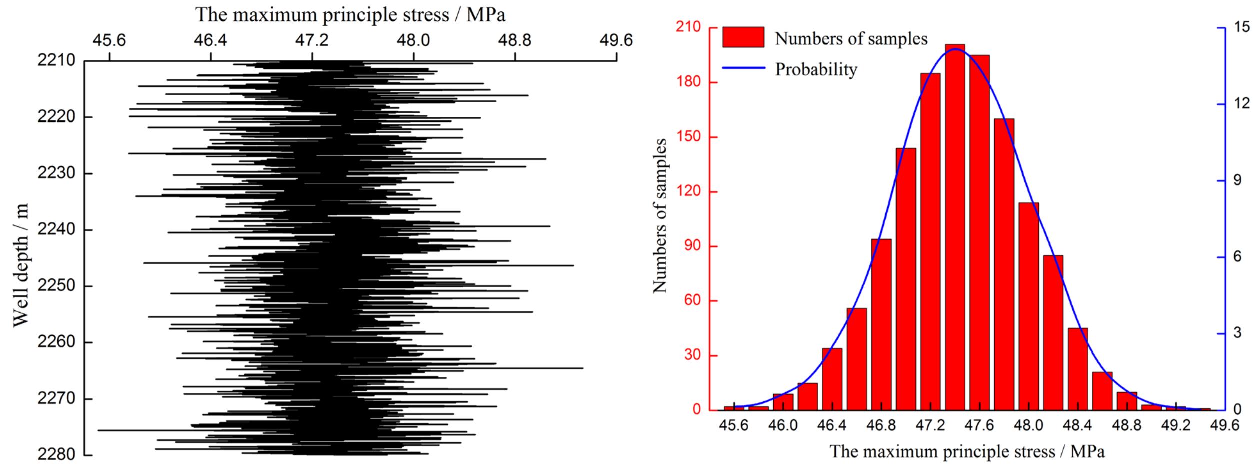


(a) Samples generation



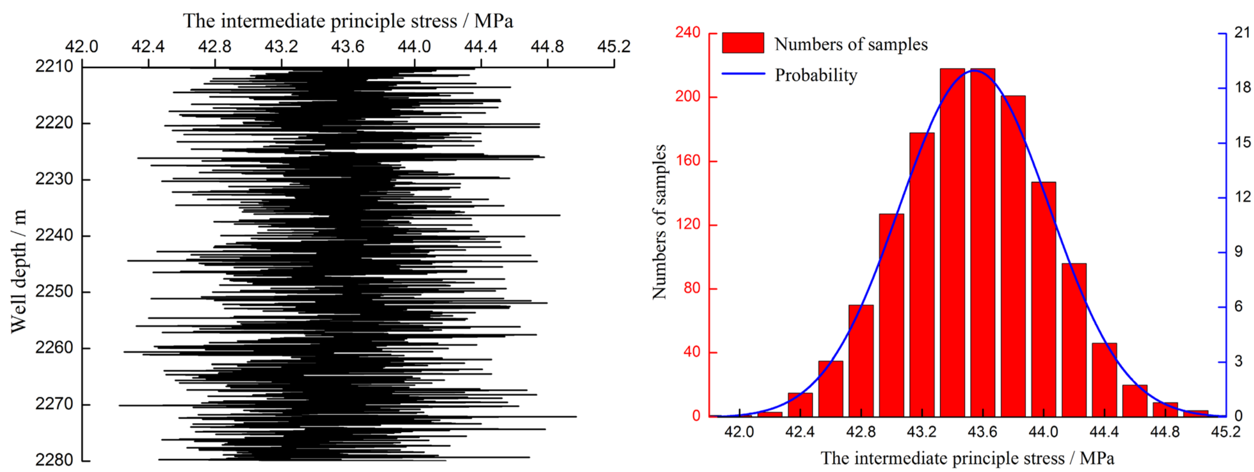
(b) Probability distribution

Fig. 1 Uncertainty quantification for the formation pressure



(a) Samples generation **(b) Probability distribution**

Fig. 2 Uncertainty quantification for the maximum principle stress



(a) Samples generation **(b) Probability distribution**

Fig. 3 Uncertainty quantification for the intermediate principal stress

4 Results and Discussion

4.1 Weight Coefficient Analysis Based on AHP

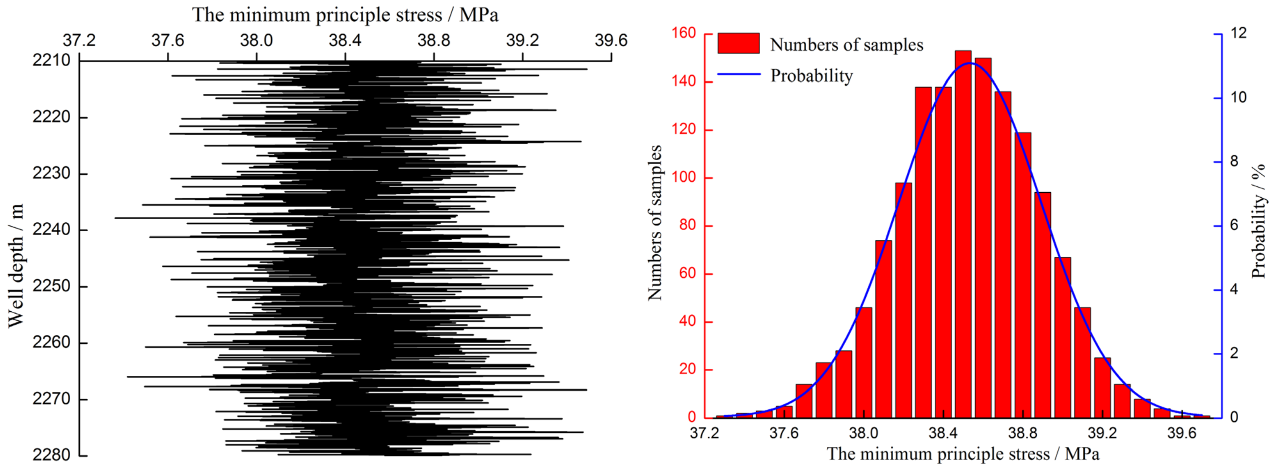
To obtain the weight coefficient, AHP created first a pairwise comparison matrix **A**, seeing Eq. (32).

$$\begin{array}{cc} \mathbf{A} & \begin{array}{cccc} \text{Mohr-C} & \text{D-P} & \text{Lade} & \text{Mogi-C} \end{array} \\ \begin{array}{c} \text{Mohr-C} \\ \text{D-P} \\ \text{Lade} \\ \text{Mogi-C} \end{array} & \begin{bmatrix} 1 & 7 & 6 & 1/4 \\ 1/7 & 1 & 1/2 & 1/8 \\ 1/6 & 2 & 1 & 1/7 \\ 4 & 8 & 7 & 1 \end{bmatrix} \end{array} \tag{32}$$

Then, weight coefficients were determined based on matrix **A**, equaling [0.29, 0.05, 0.08, 0.58], i.e., $\eta_1 = 0.29$, $\eta_2 = 0.05$, $\eta_3 = 0.08$, $\eta_4 = 0.58$. To enable the effectiveness of the weight coefficient, a consistency was checked by calculating the ratio of Consistency Index (CI)-to-Random Index (RI). The result shows that the ratio is identical to 0.089, less than 0.1, and the consistency is satisfied.

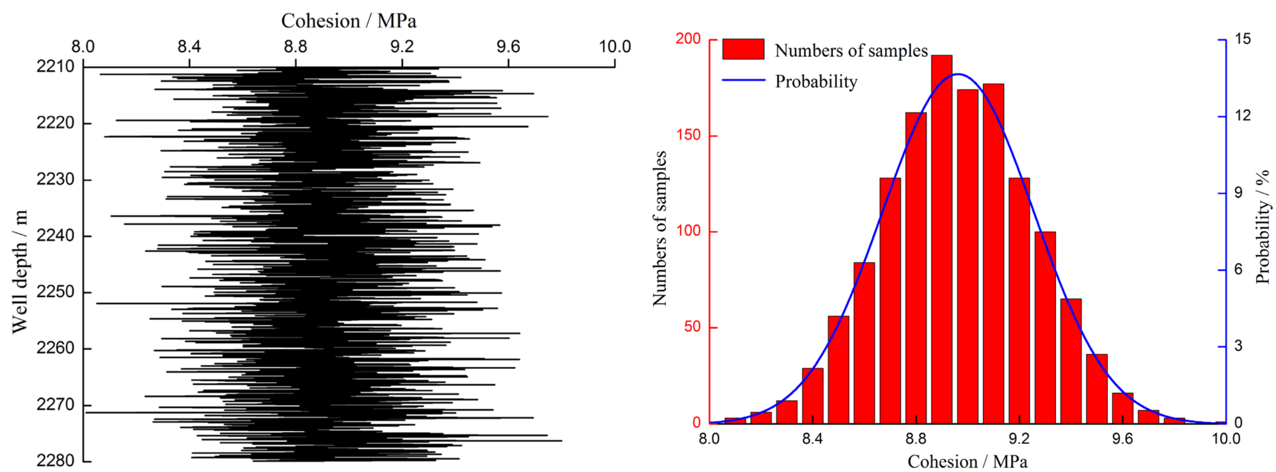
4.2 Verification of Analytical Method

Due to the uncertainty of input variables, the collapse pressure can be obtained as the probability distribution based on the analytical method, using the new collapse pressure model. Meanwhile, the probability distribution of the collapse pressure was calculated by Monte Carlo simulation,



(a) Samples generation (b) Probability distribution

Fig. 4 Uncertainty quantification for the minimum principle stress

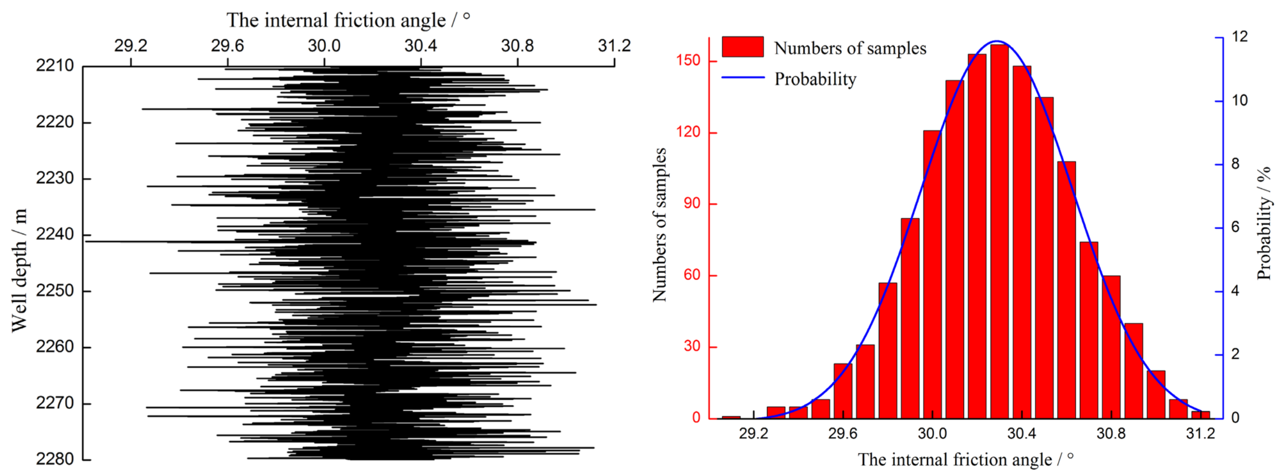


(a) Samples generation (b) Probability distribution

Fig. 5 Uncertainty quantification for the rock cohesion

with simulation times of 200,000. To verify the accuracy of the analytical method, the uncertainty results were compared for the analytical method and Monte Carlo simulation, seeing Figs. 8, 9 and 10. Especially, two sets of the weight coefficients were used in this verification, namely, $\eta_1 = 0.29$, $\eta_2 = 0.05$, $\eta_3 = 0.08$, $\eta_4 = 0.58$ and $\eta_1 = 0.3$, $\eta_2 = 0$, $\eta_3 = 0$, $\eta_4 = 0.7$, to better illustrate the generality of the proposed analytical method.

As seen from Fig. 8, the analytical result shows an agreement with Monte Carlo simulation for the property parameters of the collapse pressure, including the maximum value, the minimum value, mean, standard deviation. Additionally, Monte Carlo results of the collapse pressure were plotted in Fig. 9. Due to sufficient simulation of 200,000 times, Monte Carlo results of the collapse pressure can be treated as base values, to verify the analytical results. According to the result of Fig. 9, the collapse pressure as the random variable is satisfied to the normal distribution, which is consistent to the analytical method from distribution-type point of view. Besides, the probability distribution curves coincide well between the analytical method and Monte Carlo simulation, which can be seen in Fig. 10. Overall, the proposed analytical method has a higher accuracy to predict the uncertainty of the collapse pressure, no matter which set of weight coefficient was used in this verification.



(a) Samples generation **(b) Probability distribution**

Fig. 6 Uncertainty quantification for the internal friction angle

Table 1 Uncertainty for the input parameter

Input parameter	Maximum value	Minimum value	Uncertainty in estimation	Mean	Standard deviation	Probability distribution
Formation pressure (MPa)	24.70	23.26	+2.87%/−3.12%	24.01	0.22	Normal
Maximum principle stress (MPa)	49.33	45.51	+4.16%/−3.91%	47.36	0.55	Normal
Intermediate principle stress (MPa)	44.97	42.22	+3.33%/−2.99%	43.52	0.48	Normal
Minimum principle stress (MPa)	39.49	37.36	+2.65%/−2.89%	38.47	0.36	Normal
Internal friction angle (°)	31.12	29.01	+2.94%/−4.04%	30.23	0.33	Normal
Cohesion (MPa)	9.80	8.01	+9.86%/−10.20%	8.92	0.29	Normal

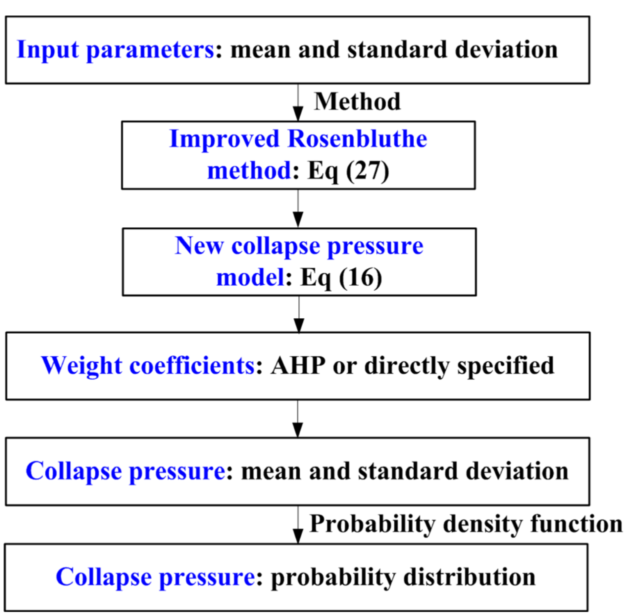


Fig. 7 A flow chart of calculations

4.3 Probability Distribution Comparison

By means of the proposed analytical method, five collapse pressure models were used to obtain the probability distribution of the collapse pressure, namely, the collapse pressure model based on Mohr–Coulomb criterion (Model I), the collapse pressure model based on Drucker–Prager criterion (Model II), the collapse pressure model based on modified Lade criterion (Model III), the collapse pressure model based on Mogi–Coulomb criterion (Model IV), and the new collapse pressure model (Model V). Especially, the weight coefficients were considered for Model V, having $\eta_1 = 0.29$, $\eta_2 = 0.05$, $\eta_3 = 0.08$, $\eta_4 = 0.58$. Figure 11 shows the probability distribution of the collapse pressure from Model I to Model V.

As seen from Fig. 11, the collapse pressure has the distribution ranges of [27.59 MPa, 32.62 MPa], [15.10 MPa, 20.42 MPa], [23.66 MPa, 28.53 MPa], [24.33 MPa, 29.28 MPa], and [24.76 MPa, 29.75 MPa] from Model I to Model V. Model I has the maximum value of the collapse

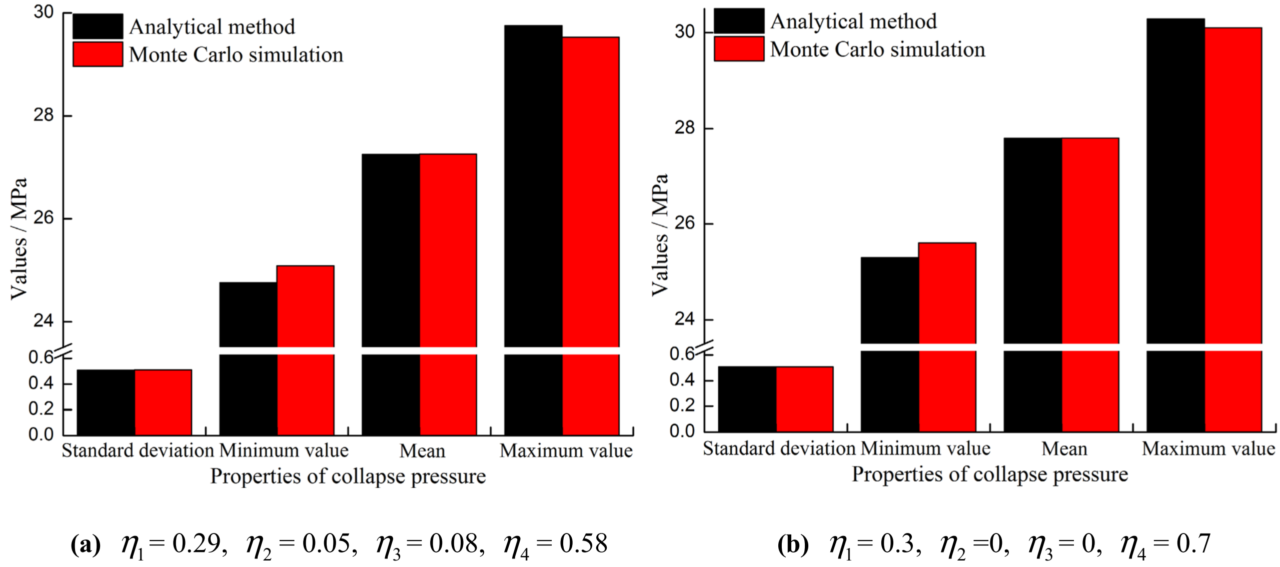


Fig. 8 Comparison of properties of collapse pressure

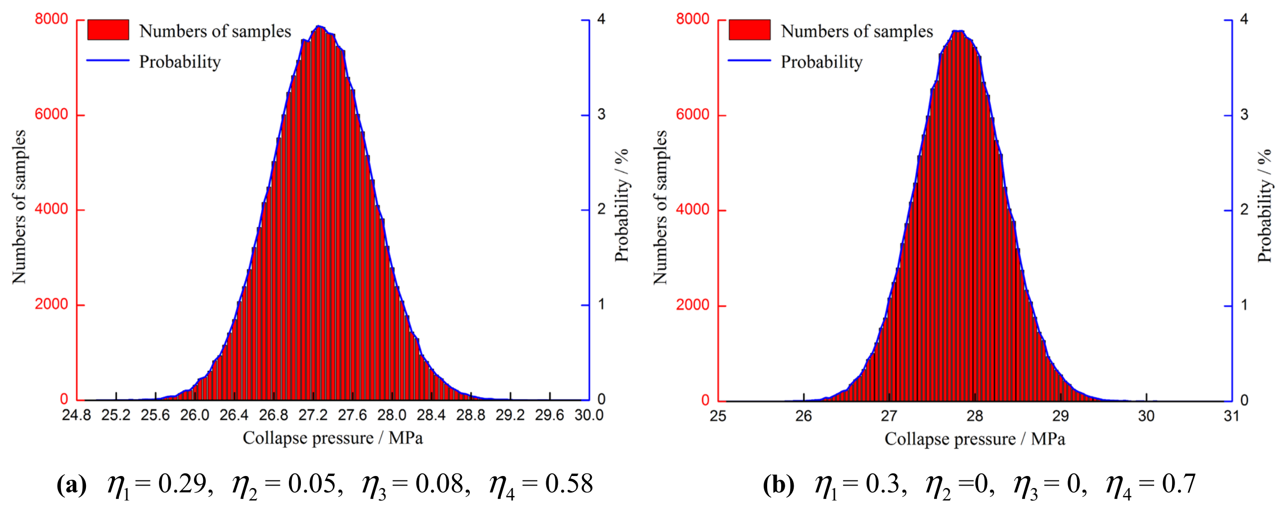
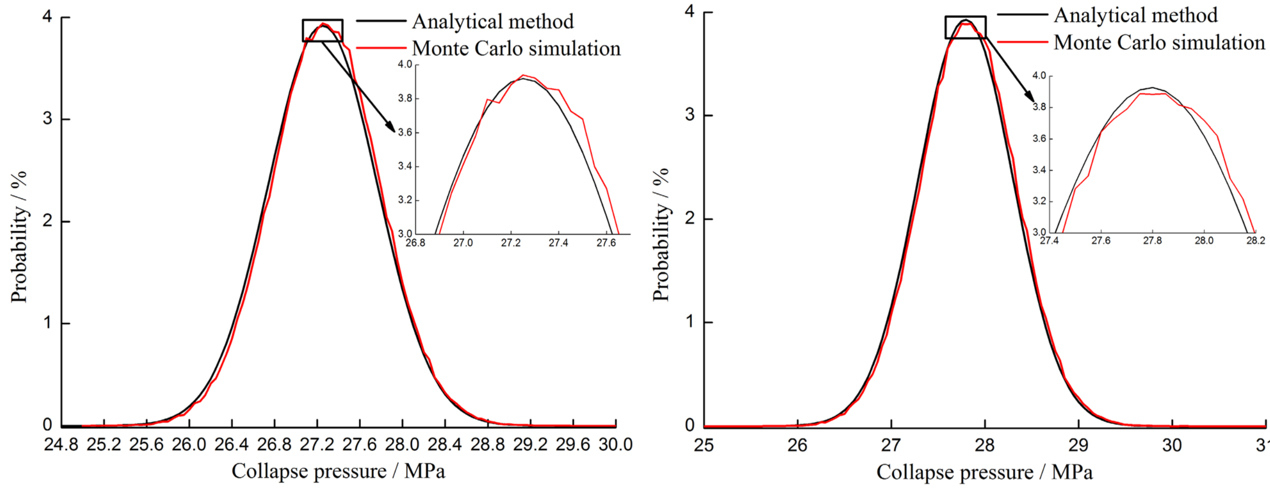


Fig. 9 Monte Carlo simulation of collapse pressure

pressure, followed by Model V, Model IV, Model III, while Model II under-predicts the collapse pressure most obviously in all the models. Model I neglects the effect of the intermediate principle stress on the rock failure, making that the collapse pressure is much higher than the ones of other models. Model V shows the closest distribution to Model IV, mainly because Model V contains a weight coefficient that reaches up to 0.58 from Mogi–Coulomb criterion. Comparing Fig. 11a–e, the probability distribution has a large difference for the collapse pressure from Model I to Model V, which means that screening the collapse pressure model is vital for predicting the probability distribution of the collapse pressure. Additionally, mean values and standard deviations of Model I–Model V are shown in Fig. 11f. The results indicate that standard deviations are approximately equal, although mean values have a large variation from Model I to Model V. In this paper, Model V was recommended to calculate the probability distribution of the collapse pressure, due to its advantageous synergies. The following section would put an emphasis on discussing the advantageous synergies of Model V.

4.4 Advantageous Synergies of Model V

According to the result of Sect. 4.3, the probability distribution of the collapse pressure has a large difference for



(a) $\eta_1 = 0.29,\ \eta_2 = 0.05,\ \eta_3 = 0.08,\ \eta_4 = 0.58$ **(b)** $\eta_1 = 0.3,\ \eta_2 = 0,\ \eta_3 = 0,\ \eta_4 = 0.7$

Fig. 10 Comparison of probability distribution for collapse pressure

Model I–Model IV. This means that Model I–Model IV may be not precise enough, even though these models have respective advantages in predicting the collapse pressure. In view of this, Model V introduced the weight coefficient to combine the advantages of Model I–Model IV, which can be treated as advantageous synergies of Model V. In Model V, the weight coefficients represent the contributions of Mohr–Coulomb, Drucker–Prager, modified Lade and Mogi–Coulomb criteria on the collapse pressure. From this point of view, Model V indeed acquires the advantageous synergies of different strength criteria. Model V can provide the probability distribution of the collapse pressure that is closest to the desired one, as long as the weight coefficient given is reasonable.

For a hypothetical well, the collapse pressure was expected with a mean of 27.20 MPa and a standard deviation of 0.544 MPa, when uncertainties for the input parameters were same as Table 1. Then, the probability distribution desired can be obtained for the hypothetical well, as shown in Fig. 12a. Using the probability distribution desired as a basis, it can be found which model has the best probability distribution to match the desired one. The comparison of the probability distribution was shown in Fig. 12b.

As seen from Fig. 12, Model I–Model IV are difficult to obtain the probability distribution of the collapse pressure desired. This is mainly because the probability distribution of the collapse pressure is fixed for Model I–Model IV, once the collapse pressure model is determined. However, Model V can provide the probability distribution that approaches the desired one, when $\eta_1 = 0.15$, $\eta_2 = 0$, $\eta_3 = 0.15$, $\eta_4 = 0.70$. This indicates that Model V has a better probability distribution to match the desired one, as long as the weight coefficient given is reasonable. From this point of view, Model V has the advantageous synergies compared to Model I–Model IV.

Model V can lead to different probability distributions of the collapse pressure, when changing the weight coefficient. Figure 13 shows probability distributions under different weight coefficients.

In Fig. 13, “Case 1” represents “$\eta_1 = 0.10$, $\eta_2 = 0.10$, $\eta_3 = 0.10$, $\eta_4 = 0.70$”; “Case 2” represents “$\eta_1 = 0.20$, $\eta_2 = 0.10$, $\eta_3 = 0.10$, $\eta_4 = 0.60$”; “Case 3” represents “$\eta_1 = 0.30$, $\eta_2 = 0.10$, $\eta_3 = 0.10$, $\eta_4 = 0.50$”; “Case 4” represents “$\eta_1 = 0.10$, $\eta_2 = 0$, $\eta_3 = 0.10$, $\eta_4 = 0.80$”; “Case 5” represents “$\eta_1 = 0.20$, $\eta_2 = 0$, $\eta_3 = 0.10$, $\eta_4 = 0.70$”; “Case 6” represents “$\eta_1 = 0.30$, $\eta_2 = 0$, $\eta_3 = 0.10$, $\eta_4 = 0.60$”. Based on the results, the probability distribution of the collapse pressure varies, as the weight coefficient changes. In other words, the probability distribution is mainly dependent on the weight coefficient for Model V. From Case 1 to Case 3, mean values of the collapse pressure increase gradually. This is because the increase of η_1 enhances the contribution of Mohr–Coulomb criterion on the collapse pressure. Comparing “Case 1–Case 3” and “Case 4–Case 6”, mean values of the collapse pressure further increase, mainly because the decrease of η_2 reduces the effect of Drucker–Prager criterion on the collapse pressure. Note that, standard deviations remain almost constant, no matter which case was used. Overall, Model V has a more comprehensive coverage for the collapse pressure, due to the variation of the weight coefficient, which is a great advantage over Model I–Model IV.

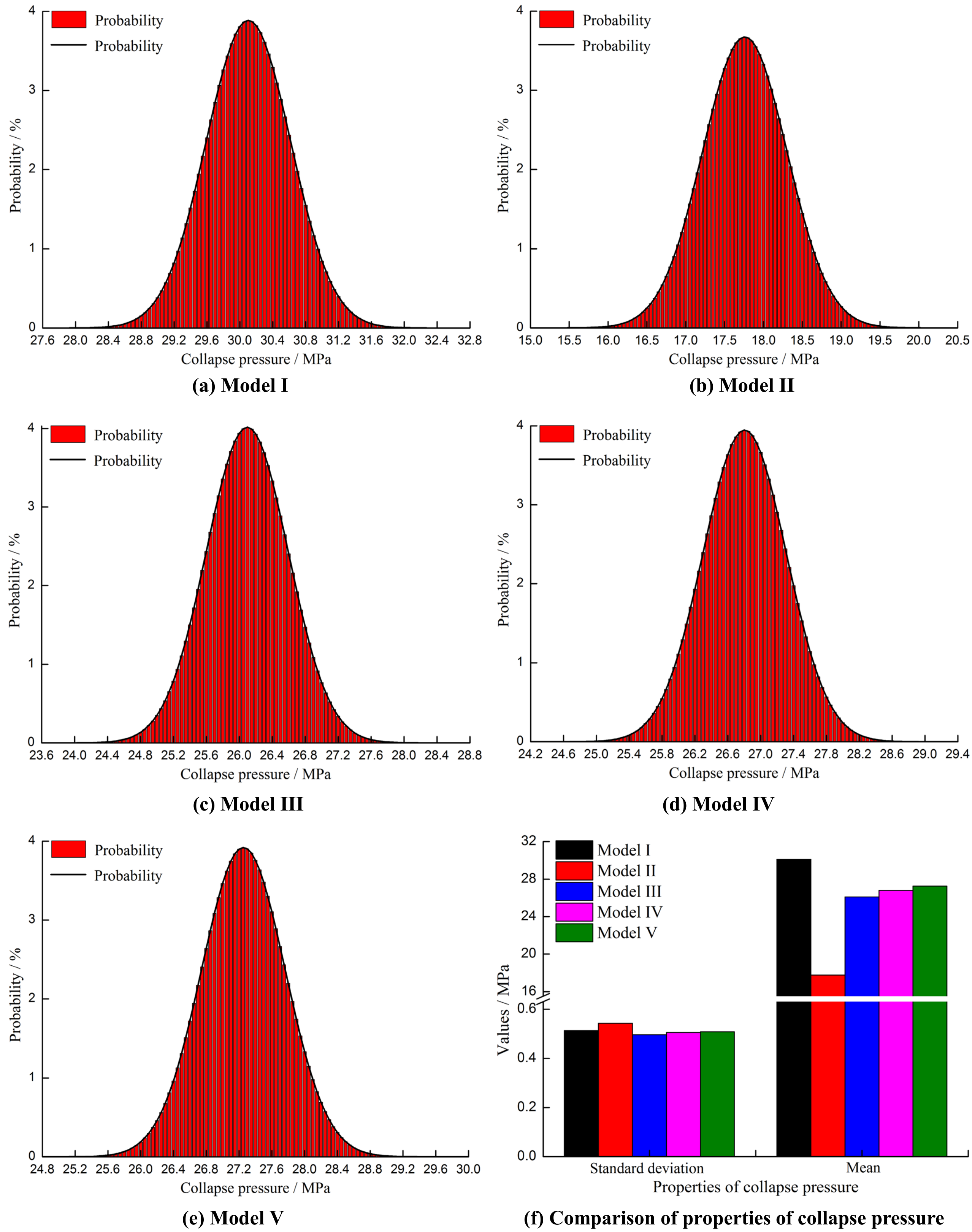
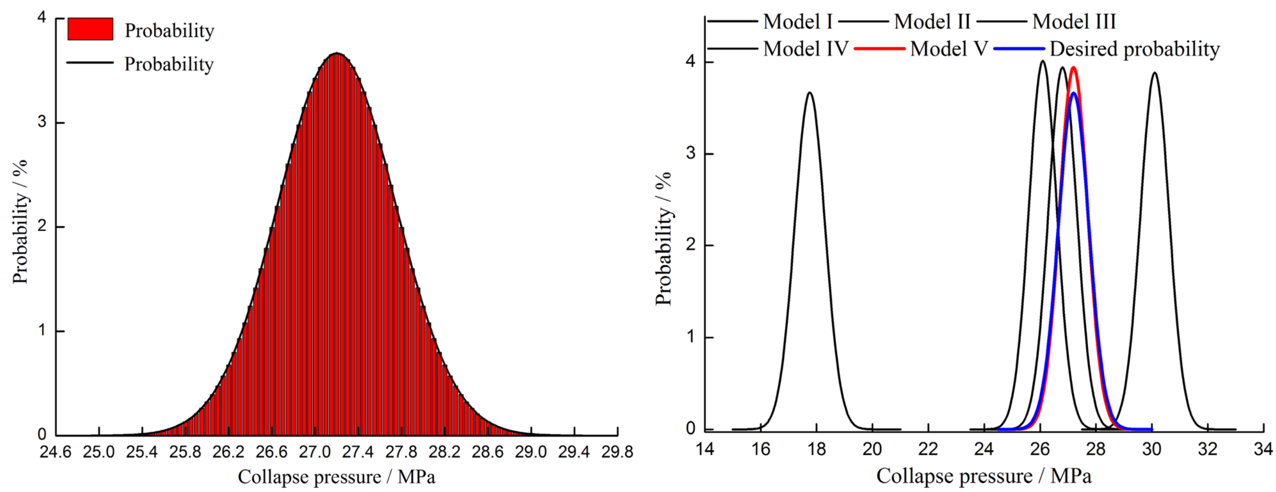
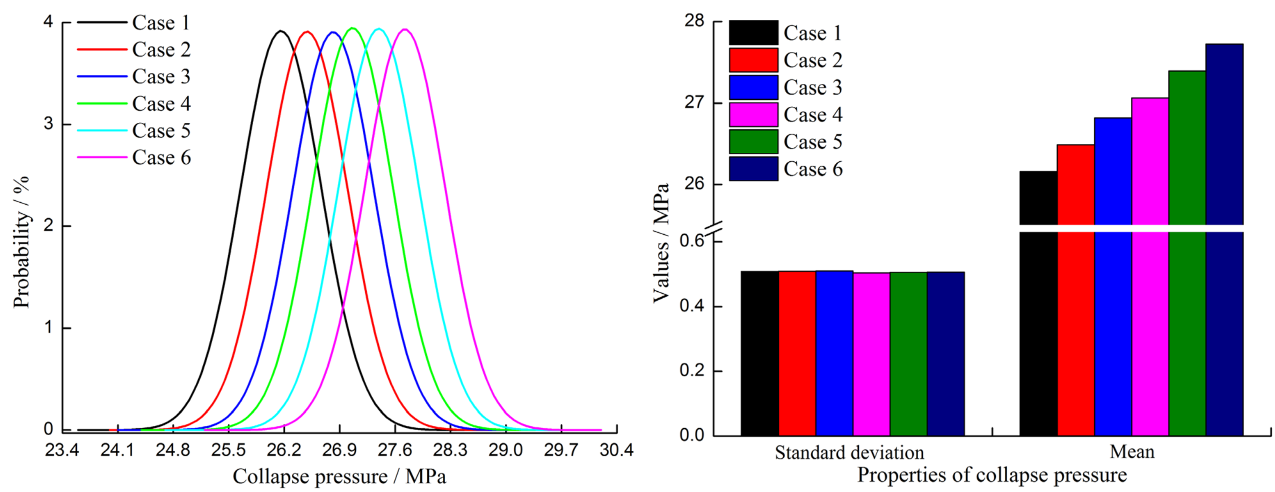


Fig. 11 Probability distribution of collapse pressure from Model I to Model V



(a) Probability distributiondesired **(b) Comparison of probability distribution**

Fig. 12 Comparison of probability distribution based on a hypothetical well



(a) Comparison of probability distribution **(b) Comparison ofmean and standard deviation**

Fig. 13 Comparison of probability distribution under different weight coefficients

5 Conclusions

1. During the uncertainty evaluation of the collapse pressure, selecting a reasonable model of the collapse pressure is very important for the prediction accuracy of the uncertainty analysis. In this paper, a new model was proposed to evaluate the wellbore collapse pressure, where the weight coefficients were introduced to represent the contributions of Mohr–Coulomb, Drucker–Prager, modified Lade and Mogi–Coulomb criteria on the collapse pressure. Especially, these weight coefficients were evaluated by analytic hierarchy process (AHP). Compared to the collapse pressure models developed, the new model shows advantageous synergies of different strength criteria.
2. Considering the new model of the collapse pressure, an analytical method was proposed to deal with the uncertainty of the collapse pressure based on Rosenbluthe method. Using the analytical model, the collapse pressure was obtained as the probability distribution. Meanwhile, the results of the probability distribution were compared between the analytical method and Monte Carlo simulation, to verify the proposed analytical method. Compared to Monte Carlo simulation, the

proposed analytical method reduces the calculation time and enhances the calculation efficiency, while maintaining a very high accuracy.

3. The probability distribution of the collapse pressure has an obvious difference using different collapse pressure models. The new collapse pressure model can provide the probability distribution that approaches the probability distribution desired as much as possible, when the weight coefficients given are suitable. Especially, the new model has a more comprehensive coverage for the collapse pressure than other models developed, due to the introduction of the weight coefficient. This can be treated as the best validation of the advantageous synergies for the new model.

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